AKADEMIE VĚD
ČESKÉ REPUBLIKY

Teze disertace
k získání vědeckého titulu "doktor věd"
ve skupině věd FYZIKÁLNĚ-MATEMATICKÝCH

MULTICHANNEL BLIND IMAGE RESTORATION
název práce

Komise pro obhajobu doktorských disertací v oboru: INFORMATIKA A KYBERNETIKA

Jméno uchazeče: ING. FILIP ŠROUBEK, PH.D.

Pracoviště uchazeče: ÚSTAV TEORIE INFORMACE A AUTOMATIZACE V.V.I, AV ČR

Místo a datum: PRAHA, 2013

Contents

Resumé

The dissertation tackles an interesting inverse problem of estimating a latent sharp image from blurred observed images. The blurring process is modeled by convolution and the inverse problem is referred to as "blind" deconvolution since we assume limited or zero knowledge of the convolution kernel. A wide range of degradation processes that occur during data acquisition can be modeled or at least well approximated by convolution. Camera/object motion, camera optics, turbulence of a measuring media such as the atmosphere are some of the examples. A frequent encounter of convolution in diverse application areas makes the deconvolution problem particularly appealing.

The dissertation consists of seven scientific articles that survey author's contribution to the theory of image deconvolution. The common framework in the presented collection is a multichannel scenario, i.e., the same scene is captured more than once and each observed image contains a slightly different convolution kernel (blur). We show that under the assumption of multichannel acquisition we have tools to estimate blurs directly from the observed images without any prior knowledge of the kernel shape. Further we show that formulating blind deconvolution as an energy minimization problem provides the necessary robustness in the case of noisy acquisitions, which is essential for usability of blind deconvolution in practical applications.

Real data seldom follow the mathematical model precisely. This is either due to unknown perturbations or the acquisition model is more complicated than the assumed mathematical model. A common problem encountered in practice is misregistration of input images. It is hard to guarantee that during multiple acquisitions the observed images will be spatially aligned. We show that the proposed multichannel blind deconvolution method automatically estimates translation among images by shifting the estimated convolution kernels in the correct direction, which makes the method robust to slight misalignment of input images.

Another common problem is that input images have insufficient spatial resolution. Increasing the image resolution is called superresolution. The aliasing effect is important in this case as the high-resolution details are recovered from the overlapping image spectra. We propose to address both deconvolution and superresolution in one common framework resulting in a blind superresolution method, which simultaneously estimates convolution kernels and the sharp image in the high-resolution domain. In practice the maximum meaningful resolution factor we can achieve (often between $2\times$ and $3\times$) is limited by the number of input images and discrepancies from the mathematical model. We show that the theory of blind superresolution derived for integer resolution factors is easily extendable to rational factors using a polyphase decomposition.

Current images have many millions of pixels and fast, close to real time,

deconvolution methods are preferred. Recent progress in the direction of fast numerical optimization methods is included in the dissertation.

The final two articles in the collection illustrate applicability of blind deconvolution in ophthalmology and mobile phone photography. Images of eye retina, analyzed by ophthalmologists are often blurred due to eye movement and pupil imperfections. We demonstrate in the first paper that multichannel blind deconvolution could be a useful tool for obtaining sharp retina images and therefore improving retina defect diagnosis. The second paper reviews our progress in implementing blind deconvolution in embedded device such as smartphones.

1 Introduction

This dissertation addresses one of the core problems of image processing, which is estimating an image from its degraded observations (measurements). Processing images becomes an every-day practice in a wide range of applications in science and technology and we rely on images with ever growing emphasis. Our understanding of the world is however limited by measuring devices that we use to acquire images. Inadequate measuring conditions together with technological limitations of the measuring devices result in acquired images that represent a degraded version of the "true" image. Fig. 1 illustrates examples of acquired images under real conditions versus ideal conditions in three different application areas. It is important to underline that the ideal conditions may not be achievable in practice and that the only solution to get the ideal image is to estimate it from the acquired ones.

The relation between the true latent image u and the degraded observed image g is given by a formula

$$g = Hu + n\,, \tag{1}$$

where H is the degradation operator and n is additive noise. By the word "degradation" we loosely mean an operator that diminishes or completely removes high frequency information (details) from images. The difficulty with H is that it is ill-conditioned, which means that during inversion noise n gets amplified and the solution is unstable. We face an *ill-posed inverse* problem that requires special handling. Our scenario is even more complicated as H is unknown, but we assume that it belongs to a certain type of degradation.

The most common type of degradation, which is considered in the dissertation, is *convolution*:

$$Hu(x) = \int h(x-t)u(t)\mathrm{d}t\,, \tag{2}$$

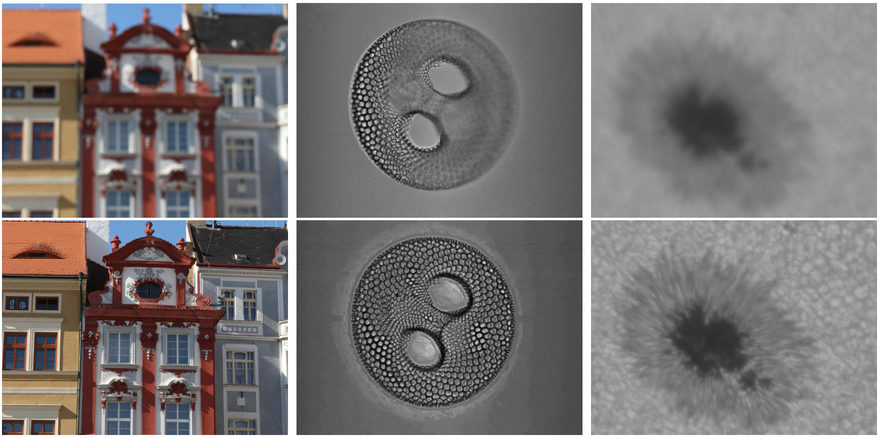


Figure 1: Examples of acquired images under degraded conditions (top row) and ideal nonviable conditions (bottom row) in three application areas: photography (house), microscopy (biological specimen), astronomy (sunspot).

where $x, t \in \mathbb{R}^2$ for images. This definition extends to any number of dimensions and not just $\mathbb{R}^2$. For example in confocal microscopy, convolution is in $\mathbb{R}^3$. Function h is a convolution kernel (or simply blur) and defines the behavior of the convolution operator. It is also called a *point spread function* (PSF), because h is an image the device would acquire after measuring an ideal point source $\delta(x)$ (delta function). Image blur due to camera motion or improper camera focus setting can be modeled by convolution. The degree of blurring influences the PSF size and the physical nature of blurring determines the PSF shape. For example, out-of-focus camera lens causes convolution with a cylindrical PSF, or camera motion causes convolution with a curvy PSF, where the curve shape is related to the trajectory of the motion; see Fig. 2. There is a wide range of imaging devices, in which the acquisition process can be modeled by convolution. Apart from devices with classical optical systems, such as digital cameras, optical microscopes or telescopes, convolution degradation occurs also in atomic force microscopy (AFM) or scanning tunneling microscopy (STM), where the PSF shape is related to the measuring tip shape. Media turbulence (e.g. atmosphere for terrestrial telescopes) can cause blurring that can be modeled by convolution, and there are many more examples. To make convolution more general, it is often necessary to allow the PSF to change over the image. In (2), h becomes a function of x as well, i.e. $h(x, t)$. This is called *space-variant* convolution, though strictly speaking it is not mathematical convolution any more. Using space-variant convolution we can model more general degradations, such as blur induced by complex

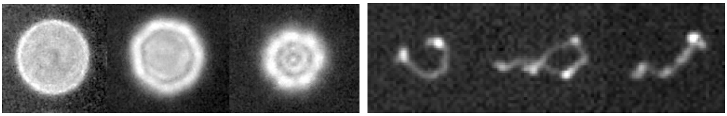


Figure 2: Examples of real camera blurs: (left) three blurs caused by out-of-focus lens with different lens parameters (focal length and aperture size), notice polygonal shape clearly visible in the central image, which corresponds to the aperture opening of 7-blade diaphragm; (right) three blurs caused by camera motion during exposure.

camera motion and rotation, out-of-focus blur in a wide-depth scene, or blur due to hot-air turbulence. Volumes acquired by a confocal microscope are in general degraded by 3D space-variant blur, which renders this particular problem even more challenging.

Deconvolution, as the name suggests, refers to the process of inverting the convolution operator H. *Blind deconvolution* denotes the case when the PSF is also unknown. If only one image g is observed then we call this problem *single-channel* blind deconvolution.

We can have more observations of the latent image u and write

$$g_k = H_k u + n_k \,, \tag{3}$$

where g_k is the k-th degraded image. Notice that the degradation operator can be different, i.e. PSF h_k is different for different k. Indeed it is highly desirable that the PSFs differ, since then multiple observations may convey complementary information. The estimation of the latent image u from the multiple observations g_k's without any knowledge of h_k's is referred to as *multichannel blind deconvolution* and this is the main topic of the selected publications, which the dissertation consists of.

An interesting extension of the above degradation operator, which is also discussed in the dissertation, is to consider in addition to convolution H a decimation operator D and rewrite model (3) as

$$g_k = DH_k u + n_k \,. \tag{4}$$

The decimation operator D models sampling on a camera sensor, which is affected by diffraction, shape of light sensitive elements and void spaces between the elements. Including D in the model allows us to increase spatial resolution of images. The corresponding inverse problem is called *superresolution*.

Section 2 overviews the evolution of state of the art in the last 10 years during which the author of the dissertation has contributed to the field of image restoration. This section discusses the development in a slightly wider perspective than the dissertation topic and includes also references to author's research articled that are not part of the dissertation collection.

Section 3 lists published research articles that make up the dissertation and for each article gives a brief overview of the main ideas and contribution to the state of the art.

2 State of the art

Recovering u from g even in the nonblind case is not straightforward. A standard technique is to convert the deconvolution problem to energy minimization [1]. The core term in the energy function implied by the model (1) is called a data-fitting or fidelity term and takes the form

$$E(u) = \|g - Hu\|_2^2 \,, \tag{5}$$

where $\|\cdot\|_p$ denotes the L_p norm. In this case, finding the minimum of $\hat{u} = \arg\min_u E(u)$ is equivalent to a lease-square fit. The difficulty of finding the minimum of (5) resides in the degradation operator H. Since blurring diminishes high frequency information (image details), the spectrum of H contains zeros or values close to zero. Therefore, H is generally not invertible. To overcome this, the idea is to regularize the problem by considering a related problem that admits a unique solution.

A classical way to solve ill-posed minimization problems is to add regularization terms. Regularization conveys additional prior knowledge of the original image u to the energy function. Priors are application dependent and general rules for constructing the priors are hard to find. Nevertheless, studying image statistics shows that the majority of natural images contain smooth regions with abrupt changes of intensity values at object boundaries that correspond to edges. An image gradient is a useful feature, which can distinguish between edges and smooth regions. Therefore, regularization terms are often functions of $\nabla u = [u_{x_1}, u_{x_2}]$. The L_p norm for $p \leq 1$ of the image gradient (a special case of $p = 1$ is called *total variation* [2]), is a popular choice for the image regularization term. Then the regularized energy becomes

$$E(u) = \|g - Hu\|_2^2 + \lambda \|\nabla u\|_p^p \,. \tag{6}$$

Parameter λ is a positive weighting constant. The first term forces the solution to be close to the observed data and the second one guarantees that the solution is sufficiently smooth in the L_p norm sense. Noise is removed in smooth regions, while edges are not excessively penalized, since we use L_p norm for $p \leq 1$ instead of L_2 norm.

In blind image restoration, the energy E becomes a functional of two unknowns, original image u and degradation H. In our case of convolution,

H is parametrized by PSF h. If both u and h are unknown, the problem is underdetermined and some additional information (e.g. regularization of h) or different minimization strategy is necessary. For example, so called "no-blur" solution, when $\hat{u} = g$ and $\hat{h} = \delta$, is one of the unwanted solutions, which gives the blurred input as the latent image and the delta function as the estimated PSF. Many commonly used energy functions get trapped in "no-blur" solution.

Another standard formulation of the image restoration problem is stochastic [3] assuming that the images and PSFs are random vector fields [4] with known prior probability distribution functions $p(u)$ and $p(h)$, respectively. The Bayesian paradigm dictates that the inference on the latent image and PSF should be based on the posterior probability

$$p(u, h|g) \propto p(g|u, h)p(u)p(h) \,, \tag{7}$$

where u and h are assumed to be independent. The conditional distribution $p(g|u, h)$ is given by our model (1). Suppose that n is white Gaussian noise then the logarithm of the conditional distribution is equivalent to (5). Different noise distributions result in different data-fitting terms. For example, the Laplace distribution implies $E(u) = \|z - Hu\|_1$. Estimating the pair $(\hat{u}, \hat{h})$ is equivalent to maximizing the posterior $p(u, h|g)$, which is commonly referred to as the maximum *a posteriori* (MAP) approach. Note that maximization of the posterior is equivalent to minimization of $-\log p(u, h|g)$, which is an energy minimization task (6), where the priors play a role of regularization terms. The simplest but also the most common method maximizes the posterior in an alternative manner with respect to u and h. Unfortunately, the posterior has very uneven shape with many local peaks and alternating maximization often returns an incorrect solution.

In the case of single-channel blind deconvolution, proposed approaches include stronger regularization both on the image and blur and above all must use more sophisticated estimation procedures as discussed in Sec. 2.1. The multichannel case discussed in Sec. 2.2 permits estimation of the blurs without any prior knowledge of their shape. The space-variant case with parametric approaches is covered in Sec. 2.3.

2.1 Single-channel blind deconvolution

One way to tackle the problem, when we have only one observation and no knowledge of the PSF, is to assume a parametric model of the PSF and search in the space of parameters and not in the full space of PSFs. Chang *et al.* in [5] investigated zero patterns of the Fourier transform or cepstrum, and assumed only parametric motion or out-of-focus blurs. More low-level parametric methods for estimating general motion blurs were proposed in [6, 7, 8, 9]. Parametric methods have two disadvantages. They are more restrictive than the fully blind ones and they can also be computationally more demanding. Even if minimization with respect to the unknown PSF is linear, minimization with respect to one of the parameters of the PSF does not have to be linear and thus effective methods for solving linear problems can not be applied. Real PSFs always differ slightly from their parametric models and this prevents the parametric methods to find an exact solution.

There has been a considerable effort in the image processing community in the last three decades to find a reliable algorithm for single-channel blind deconvolution. First algorithms appeared in telecommunication and signal processing in early 80's [10]. For a long time, the problem seemed too difficult to be solved for complex blur kernels. Proposed algorithms usually worked only for special cases, such as astronomical images with uniform (black) background, and their performance depended on initial estimates of PSFs; see [11, 12, 13].

Over the last few years, single-channel blind deconvolution experiences a renaissance. The key idea of new algorithms is to address the ill-posedness of blind deconvolution by characterizing the prior $p(u)$ using natural image statistics and by a better choice of estimators. The idea of natural image statistics was recently explored by the author of the dissertation in [14]. A heated activity started with the work of Fergus *et al.* [15], who applied variational Bayes to approximate the posterior $p(u,h|g)$ by a simpler distribution $q(u,h) = q(u)q(h)$. Other authors [16, 17, 18, 19] stick to the "good old" alternating MAP approach, but by using ad hoc steps, which often lack rigorous explanation, they converge to a correct solution. Levin *et al.* in [20, 21] proved that a proper estimator matters more than the shape of priors. They showed that marginalizing the posterior with respect to the latent image u leads to the correct solution of the PSF h. The marginalized probability $p(h|g)$ can be expressed in a closed form only for simple priors that are, e.g., Gaussian. Otherwise approximation methods such as Variational Bayes [22] or the Laplace approximation [23] must be used.

2.2 Multichannel blind deconvolution

The framework of multiple observations as defined in (3) provides the necessary constraint to make the image restoration task well posed. One of the earliest intrinsic multichannel blind deconvolution methods [24] was designed particularly for images blurred by atmospheric turbulence. Harikumar *et al.* [25] proposed an indirect algorithm, which first estimates the blur functions and then recovers the original image by standard nonblind methods. The blur functions are equal to the minimum eigenvector of a special matrix constructed from the blurred images, which is the same idea published earlier for 1D signals in [26]. Necessary assumptions for perfect recovery of the blur functions are noise-free environment and channel coprimeness, i.e. a scalar constant is the only common factor of the blurs. Giannakis *et al.* [27] developed another indirect algorithm based on Bezout's identity of coprime polynomials which finds restoration filters and by convolving the filters with the observed images recovers the original image. Both algorithms are vulnerable to noise and even for a moderate noise level restoration may break down. In the latter case, noise amplification can be attenuated to a certain extent by increasing the restoration filter order, which comes at the expense of deblurring. Pai *et al.* [28] suggested two multichannel restoration algorithms that estimate directly the original image from the null space or from the range of a special matrix. Another direct method based on the greatest common divisor was proposed in [29]. Interesting approaches based on the ARMA (autoregressive moving average) model are given in [30]. Multichannel blind deconvolution based on the Bussgang algorithm was proposed in [31], which performs well on spatially uncorrelated data, such as binary text images and spiky images. Most of the algorithms lack the necessary robustness since they do not include any noise assumptions in their derivation and miss regularization terms. The author of the dissertation proposed an iterative multichannel algorithm [32] that performs well even on noisy images. It is based on least-squares deconvolution by anisotropic regularization of the image and between-channel regularization of the blurs. Another drawback of the multichannel methods is that the observed images must be spatially aligned, which is seldom true. A first attempt in this direction was done by the author in [33], where blind deconvolution of images that are mutually shifted by unknown vectors was proposed. The author extended this idea to superresolution in [34]. In superresolution, the physical resolution of the image is increased, which is equivalent to considering both convolution and decimation as in (4).

Blind deconvolution in the multichannel framework is in general a well-posed inverse problem. However, in many practical situations we do not have multiple observation of the same scene, which would differ only by the convolution kernel, and we must revert to the single-channel case.

2.3 Space-variant blind deconvolution

Space-variant blind deconvolution is even more complicated as the PSF is also a function of the position vector. As a rule, the space-variant PSF cannot be expressed by an explicit formula but in many cases it has a special structure that can be exploited. For example, the blur caused by camera rotation is limited by three degrees of freedom of rigid body rotation. If we have an estimate of the camera rotation from inertial sensors [35] or other sources [36, 37, 38], we are able to reconstruct the PSF and recover the latent image. Unfortunately, in practice, the PSF must be estimated directly from the input images. If only one type of blur source is considered (e.g. rotation), we can express the degradation operator as a linear combination of basis blurs (or images) and solve the blind problem in the space of the basis, which has much lower dimension than the original problem. Whyte *et al.* [39] considered rotations about three axes up to several degrees and described blurring using three basis vectors. For blind deconvolution, they used an algorithm analogous to [15] based on marginalization over the latent sharp image. Gupta et al. [40] and Hirsch et al. [41] adopted a similar approach, replacing rotations about x and y axes by translations. Removing out-of-focus blur is a more complex problem, since the PSF depends on object distance and we need to estimate also the depth map as was proposed in [42, 43].

If the PSF changes smoothly over the image, the PSFs can be considered as constant on a small neighborhood and estimated on a regularly spaced grid. This idea has been proposed by the author of the dissertation in [44] and later extended to superresolution in [45]. For estimation, we can apply locally single-channel blind deconvolution methods [46, 47], or if a pair of blurred and noisy/underexposed images is available, multichannel methods [48, 49]. Recently, more accurate parametric interpolation of PSFs on the grid has been proposed by the author in [50].

An especially difficult situation is that of the blur caused by object motion, as objects usually move independently of each other and often in different directions. In order to achieve a good quality of deblurring, the object must be precisely segmented, taking into account partial occlusion close to object outline. Most of the methods [51, 52, 53] follow the pioneering paper of Levin [54] that assumed that objects move with a constant velocity and segmented objects based on a statistics of image derivatives. A completely novel method without the need to segment objects was recently proposed in [55].

3 Research articles in the dissertation

The dissertation is a collection of seven research articles that have one common theme of multichannel blind restoration. The acquisition model is assumed to be of the form (3) and in all the cases except one (paper no. 2) the energy minimization approach is considered. The first five articles are chronologically sorted and summarize contribution of the author to the theory of blind deconvolution. The remaining two articles are examples of applied research articles that illustrate the use of blind deconvolution in practice.

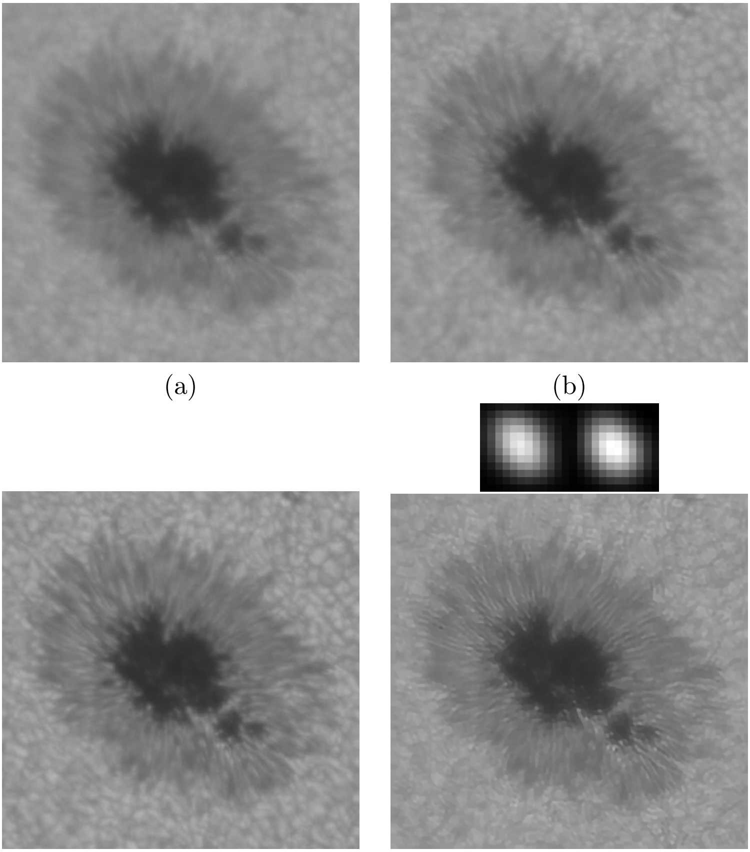
1. F. Šroubek and J. Flusser, "Multichannel blind iterative image restoration," *IEEE Transactions on Image Processing*, vol. 12, no. 9, pp. 1094–1106, 2003.

   This research article was the first step in the direction of robust multichannel deconvolution methods. We use energy minimization approach with regularization. The PSF regularization is based on a simple but elegant idea presented originally by Harikumar *et al.* in [25]. They showed that by constructing a special matrix from blurred input images, we can determine PSFs as minimum eigenvalues of the matrix. However, stability of Harikumar's method deteriorates quickly with increasing noise. Instead of using the special matrix directly, we construct from the matrix a quadratic regularization term, which is intrinsically multichannel as it couples all the input images and approaches the minimum for correct PSFs. Then we minimize the regularized energy function with respect to the image and PSFs. To increase stability even further we include image regularization based on image gradients, such as Total Variation [56] or Mumford-Shah functional [57]. The final energy function is convex but the solution leads to nonlinear equations. This drawback is solved by a half-quadratic algorithm [58], which converts the problem to a set of linear equations. A special attention is paid to discretization of image regularization terms using four-connectivity and eight-connectivity approximation. The performance of the proposed method is evaluated on synthetically blurred data and also on camera out-of-focus images and astronomical data; see an example in Fig. 3.

The main contribution of this research article is in constructing a novel multichannel regularization term and proposing an iterative method for blind deconvolution, which is robust to noise and thus suitable for practical applications.

2. F. Šroubek and J. Flusser, "Multichannel blind deconvolution of spatially misaligned images," *IEEE Transactions on Image Processing*, vol. 14, no. 7, pp. 874–883, 2005.

   In this research article, we adopt a stochastic approach to multichannel blind deconvolution and formulate the restoration problem as a MAP inference; see (7). Regularization terms are now replaced by prior distributions of images and blurs. This interpretation introduces covariance matrices that were omitted in the original formulation and allows us to better understand meaning of weighting parameters in front of these terms. We also prove that the proposed method can compensate for a misalignment of input blurred images. From the practical point of view, this is an important feature. We require multiple (minimum two) images of the same scene that are blurred in a slightly different way. Video sequences or continuous shooting in digital cameras often provide data where neighboring frames (images) depict the same scene with blurs slightly varying in time. However, such images are rarely spatially aligned (registered). We can use registration methods in [59] to geometrically align input blurred images, but registration of blurred



(a) (b) (c) (d)

Figure 3: Multichannel blind deconvolution of astronomical data: A sunspot is observed with a terrestrial telescope and a sequence of images is recorded. Due to atmospheric turbulence the observations are blurred. Images (a) and (b) show two blurred observations from the sequence, which are deconvolved in (d) with the proposed algorithm (estimated PSFs are on the top). Image (c) is the least degraded observation from the sequence and compare to the reconstructed image (d) it is still considerably blurred.

images is imprecise. We show that by overestimating the blur support, the proposed method is able to automatically shift the estimated blurs and thus cancel spatial misalignment of the images as seen in Fig. 4.

The main contribution of this research article is in built-in compensation for misalignment of input images, which further increases applicability of the proposed method.

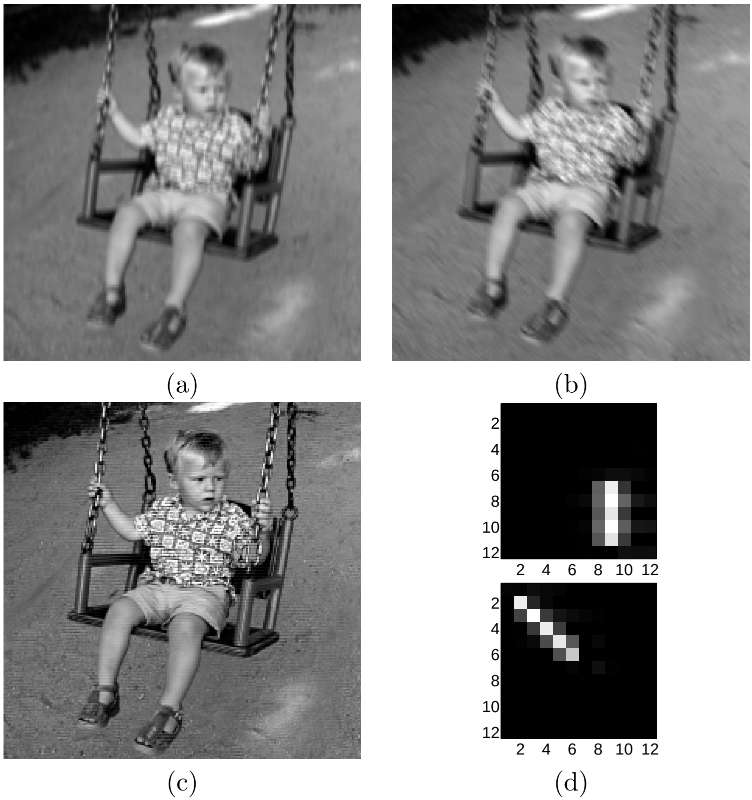


Figure 4: Robustness to misalignment: Two synthetically blurred images (a) and (b) were shifted by an unknown number of pixels and deconvolved in (c) with the proposed algorithm. Note that the estimated PSFs in (d) are automatically shifted to compensate for misalignment in the input data.

3. F. Šroubek, G. Cristóbal, and J. Flusser, “A unified approach to superresolution and multichannel blind deconvolution,” *IEEE Transactions on Image Processing*, vol. 16, no. 9, pp. 2322–2332, 2007.

   In the previous two research articles, we have developed a theory of multichannel blind deconvolution. Multiple observations of the same scene give us one additional benefit. If the observations differ by subpixel shifts we can also increase spatial resolution of the latent image (superresolution), which is explored in this research article. We assume the multichannel acquisition model with decimation as defined in (4).

We prove that even in the presence of the decimation operator D, which does not commute with the convolution operator H, we can construct a blur regularization term, which is similar to the regularization term in the classical multichannel blind deconvolution problem. The regularization term is not strictly convex and approaches the minimum on a subspace of dimensions proportional to the superresolution factor. The superresolution factor is a user parameter and determines the final high resolution of the latent image. With an increasing superresolution factor the minimum number of input images necessary to construct the regularization term increases proportionally. The regularized energy function is minimized with respect to the image and blurs as in the case of multichannel blind deconvolution. However in this case we also recover the lost spatial resolution of the latent image. We named this problem blind superresolution. An example of superresolution performance is in Fig. 5.

The proposed blind superresolution method went way beyond standard superresolution techniques. While estimating the blurs in the high resolution grid of the final latent image, we calculate not only PSFs but also subpixel shifts. This made it one of the first methods that performs deconvolution and resolution enhancement simultaneously.

(a) (b) (c)

Figure 5: Superresolution of images acquired with a digital camera: Several low-resolution images were acquired with the digital camera and superresolution has been applied. (a) bilinear interpolation of one low-resolution image, (b) result of the proposed superresolution algorithm, (c) image taken by the same camera but with optical zoom. The proposed algorithm achieves reconstruction comparable to the image with optical zoom.

4. F. Šroubek, J. Flusser, and G. Cristobal, “Super-resolution and blind deconvolution for rational factors with an application to color images,” *Computer Journal*, vol. 52, no. 1, pp. 142–152, 2009.

   The previous research article demonstrates that superresolution neatly combines with multichannel blind deconvolution. The minimum number of input images required for well-posed blind superresolution depends on the superresolution factor. For example estimating blurs and in-

input 1.25 1.5 1.75 2.0 2.5 3.0

Figure 6: Superresolution with non-integer factors of short-exposure images. The first left image is one of ten low-resolution frames acquired by a webcamera that were used to estimate high-resolution images. The proposed method was run with different superresolution factors from 1.25 to 3. The estimated HR images appear in their original size.

creasing resolution by a factor of 2, requires at least 5 images. If the factor is 3, we already need 10 images. In many practical applications it is difficult to guarantee this minimum number. In addition, acquired images do not follow precisely our mathematical model, which implies that superresolution factor of more than 2 provides negligible improvement in practice as was experimentally demonstrated. These facts show that non-integer superresolution factors below two are meaningful as they require less number of input images and recover high frequency information. This work uses the notion of polyphase decomposition to derive PSF regularization terms that work for any rational superresolution factor. We can thus extend conclusions derived in the previous research article about blind superresolution to factors such as $3/2 = 1.5$ (requires 3 input images) or $7/4 = 1.75$ (requires 4 input images). Other improvements discussed in the paper are image regularization terms for color images, and advantages of image registration performed in the decimation matrix versus registration done beforehand.

The main goal of this paper has been to extend the theory of blind superresolution for integer factors to rational factors. Examples of superresolution with different factors are in Fig. 6.

5. F. Šroubek and P. Milanfar, "Robust multichannel blind deconvolution via fast alternating minimization," *IEEE Transactions on Image Processing*, vol. 21, no. 4, pp. 1687–1700, 2012.

The quality of image deconvolution is very sensitive to accuracy with which the PSF is estimated. Disturbing artifacts appear in deconvolved images due to inaccurate PSF estimation. Using stronger image regularization we can avoid the artifacts but we inevitably loose details. In the first part of this research article, we analyze the multichannel blur regularization term and show that its dependence on noise may bias the estimation of PSFs in the noisy case. Using filtered images instead of the original intensity values to construct the blur regularization term diminishes the bias and improves the accuracy of PSF estimation. The second part of the paper is dedicated to a fast numerical optimization method, which would allow blind deconvolution of large images (several Mpixels) and large blurs (up to 100×100 pixels). Again we use alternating minimization between two steps: minimization with respect to the latent image and minimization with respect to the PSFs. However this time, we solve a nonlinear problem in each step by applying a variable splitting technique to convert the problem to constrained optimization and then using an augmented Lagrangian method to solve the constrained optimization. The augmented Lagrangian method is a fast converging method, which can solve the blind deconvolution problem in an efficient way. Examples of blind deconvolution of high resolution photos captured with a DSLR camera conclude the paper. One example is presented in Fig. 7.

The main contribution of this research article is in improving accuracy of PSF estimation and providing a fast and reliable multichannel blind deconvolution algorithm that copes with high-resolution image and large blurs.

6. A. Marrugo, M. Šorel, F. Šroubek, and M. Millan, "Retinal image restoration by means of blind deconvolution," *Journal of Biomedical Optics*, vol. 16, no. 11, pp. 116016-1-11, 2011.

   This applied research article is an example of direct application of multichannel blind deconvolution illustrating a step towards computer-assisted diagnosis and telemedicine in ophthalmology. Here we present a method for color retinal image restoration by means of multichannel blind deconvolution. The method is applied to a pair of retinal images acquired within a lapse of time, ranging from several minutes to months. It consists of a series of preprocessing steps to adjust the images so they comply with the considered degradation model (3), followed by the estimation of the PSF and, ultimately, image deconvolution. The preprocessing is composed of image registration, uneven illumination compensation, and segmentation of areas with structural changes. In addition, we have developed a procedure for the detection and visualization of structural changes. This enables the identification of subtle developments in the retina not caused by variation in illumination or blur. The method was tested on synthetic and real images. An illustration of algorithm's performance on real images is in Fig. 8.

   The main purpose of this paper has been to investigate a new approach for retinal image restoration based on multichannel blind deconvolution.

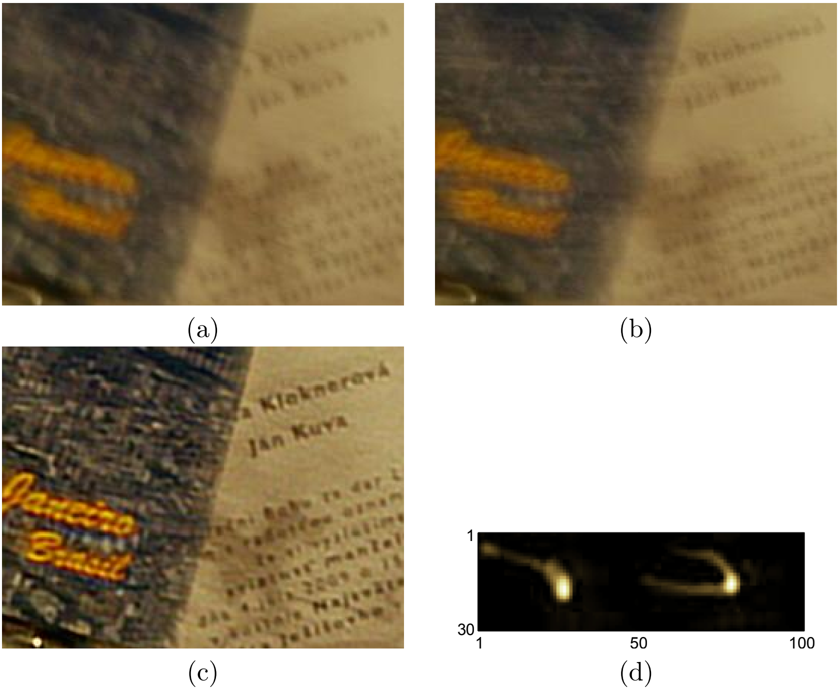
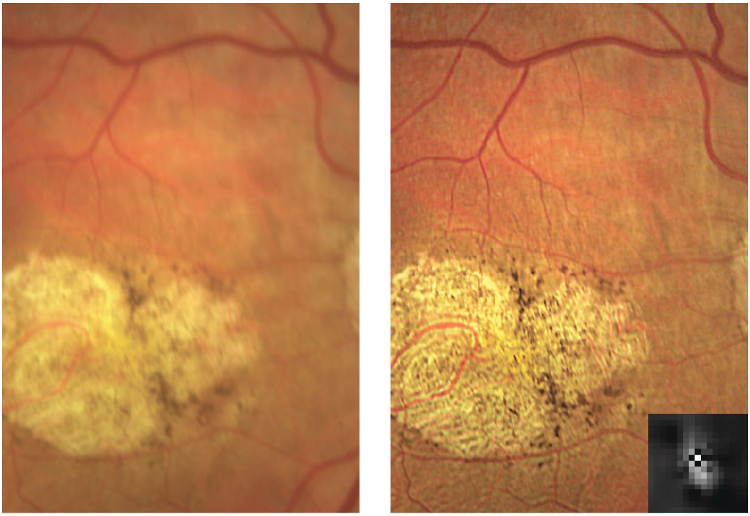


Figure 7: Fast deconvolution of photos acquired with high megapixel cameras: Two blurry images of size 2048×1536 with close-ups in (a) and (b), respectively, were processed with the proposed algorithm. The close-up of the estimated output sharp image is in (c) and the estimated large PSFs of size 50×30 are in (d).

7. O. Šindelář and F. Šroubek, "Image deblurring in smartphone devices using built-in inertial measurement sensors," *Journal of Electronic Imaging*, vol. 22, no. 1, pp. 011003-1-8, 2013.

   This is another example of an applied research article. The target application is photography on embedded devices. Blur induced by camera motion is a frequent problem in photography mainly when the light conditions are poor. As the exposure time increases, involuntary camera motion has a growing effect on the acquired image. Image stabilization devices that help to reduce the motion blur by moving the camera sensor in the opposite direction are becoming more common. However, such hardware remedy has its limitations as it can compensate only for motion of a very small extent and speed. Deblurring the image offline using mathematical algorithms is usually the only choice we have in



(a) (b)

Figure 8: Original (a) and restored retinal image (b) using the blind deconvolution method. The images are cropped to represent the region of interest given by the pathological area. The estimated PSF is shown in the bottom left corner of the restored image.

order to obtain a sharp image. Motion blur can be modeled by convolution and then the deblurring process is deconvolution. Many devices, such as modern smartphones, are now equipped with inertial sensors (gyroscopes and accelerometers) that can give us a very accurate information about camera motion. If we are able to reconstruct camera path then we can recover blur and perform nonblind image deblurring. This idea was originally described in [35] but the authors have designed an expensive measuring apparatus consisting of a DSLR camera and a set of inertial sensors, and perform image deblurring offline on a computer. Our work is based on the same idea but the aim is to show that image deblurring is feasible on modern smartphones without the requirement of other devices; see an example in Fig. 9.

The main contribution of this work is to illustrate that blur estimation with built-in inertial sensors is possible and to implement image deblurring on a smartphone, which works in practical situations and is relatively fast to be acceptable for a general user.

(a) (b)

Figure 9: Smartphone deconvolution application: (a) image acquired with the phone and blurred due to camera shake, (b) sharp image estimated by the deconvolution algorithm running on the phone using prior information about camera motion from phone gyroscopes.

4 Conclusions

The presented dissertation summarizes author's contribution to the theory of blind deconvolution in the last ten years. The underlying theme linking the collection of seven publications that comprise the dissertation is multichannel blind deconvolution. The presented research articles summarize gradual improvements in the field of blind deconvolution that resulted in a robust algorithm, which works with misaligned high-resolution blurry images, can cope with large blurs, and provides solution in short computational time. Multichannel framework of blind deconvolution is extended to resolution enhancement (superresolution), which is covered by two research articles in the collection. The applicability of the approach has been demonstrated on many practical examples. A various versions of the restoration algorithm are available for free for research purposes on the institute web pages. To date, we file over 1000 downloads of the software, which indicates a high interest of the research community in this topic. This fact is also supported by a relatively high impact of the presented collection of articles, which is close to 170 citations (according to SCOPUS) in total excluding self-citations.

References

[1] M. Banham and A. Katsaggelos, "Digital image restoration," *IEEE Signal Processing Magazine*, vol. 14, no. 2, pp. 24–41, Mar. 1997.

[2] L. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D*, vol. 60, pp. 259–268, 1992.

[3] P. Campisi and K. Egiazarian, Eds., *Blind Image Deconvolution, Theory and Application.* CRC Press, 2007.

[4] A. Rosenfeld and A. C. Kak, *Digital Picture Processing*, 2nd ed. Orlando, FL, USA: Academic Press, Inc., 1982.

[5] M. Chang, A. Tekalp, and A. Erdem, "Blur identification using the bispectrum," *IEEE Trans. Signal Processing*, vol. 39, no. 10, pp. 2323–2325, Oct. 1991.

[6] Q. Shan, W. Xiong, and J. Jia, "Rotational motion deblurring of a rigid object from a single image," in *Proc. IEEE 11th International Conference on Computer Vision ICCV 2007*, 14–21 Oct. 2007, pp. 1–8.

[7] A. Stern and N. Kopeika, "Analytical method to calculate optical transfer functions for image motion and vibrations using moments," *J. Opt. Soc. Am. A*, vol. 14, no. 2, pp. 388–396, Feb. 1997.

[8] Y. Yitzhaky and N. Kopeika, "Identification of blur parameters from motion blurred images," *Graphical Models and Image Processing*, vol. 59, no. 5, pp. 310–320, Sep. 1997.

[9] J. Oliveira, M. Figueiredo, and J. Bioucas-Dias, "Blind estimation of motion blur parameters for image deconvolution," in *Pattern Recognition and Image Analysis*, ser. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2007, vol. 4478, pp. 604–611. [Online]. Available: http://dx.doi.org/10.1007/978-3-540-72849-8_76

[10] D. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," *IEEE Transactions on Communications*, vol. 28, no. 11, pp. 1867–1875, 1980.

[11] G. Ayers and J.C.Dainty, "Iterative blind deconvolution method and its application," *Optical Letters*, vol. 13, no. 7, pp. 547–549, Jul. 1988.

[12] T. Chan and C. Wong, "Total variation blind deconvolution," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 370–375, Mar. 1998.

[13] R. Molina, J. Mateos, and A. K. Katsaggelos, "Blind deconvolution using a variational approach to parameter, image, and blur estimation," *IEEE Transactions on Image Processing*, vol. 15, no. 12, pp. 3715–3727, Dec. 2006.

[14] J. Kotera, F. Sroubek, and P. Milanfar, "Blind deconvolution using alternating maximum a posteriori estimation with heavy-tailed priors," in *Proceedings of CAIP*, 2013.

[15] R. Fergus, B. Singh, A. Hertzmann, S. T. Roweis, and W. T. Freeman, "Removing camera shake from a single photograph," in *SIGGRAPH '06: ACM SIGGRAPH 2006 Papers.* New York, NY, USA: ACM, 2006, pp. 787–794.

[16] J. Jia, "Single image motion deblurring using transparency," in *Proc. IEEE Conference on Computer Vision and Pattern Recognition CVPR '07*, 17–22 June 2007, pp. 1–8.

[17] N. Joshi, R. Szeliski, and D. J. Kriegman, "PSF estimation using sharp edge prediction," in *Proc. IEEE Conference on Computer Vision and Pattern Recognition CVPR 2008*, 23–28 June 2008, pp. 1–8.

[18] Q. Shan, J. Jia, and A. Agarwala, "High-quality motion deblurring from a single image," in *SIGGRAPH '08: ACM SIGGRAPH 2008 papers.* New York, NY, USA: ACM, 2008, pp. 1–10.

[19] L. Xu and J. Jia, "Two-phase kernel estimation for robust motion deblurring," in *Proceedings of the 11th European conference on Computer vision: Part I*, ser. ECCV'10. Berlin, Heidelberg: Springer-Verlag, 2010, pp. 157–170. [Online]. Available: http://portal.acm.org/citation.cfm?id=1886063.1886077

[20] A. Levin, Y. Weiss, F. Durand, and W. Freeman, "Understanding and evaluating blind deconvolution algorithms," in *Proc. IEEE Conference on Computer Vision and Pattern Recognition CVPR '09*, 2009, pp. 1964–1971.

[21] A. Levin, Y. Weiss, F. Durand, and W. T. Freeman, "Understanding blind deconvolution algorithms," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 12, pp. 2354–2367, 2011.

[22] J. Miskin and D. J. MacKay, "Ensemble learning for blind image separation and deconvolution." in *Advances in Independent Component Analysis*, M. Girolani, Ed. Springer-Verlag, 2000, pp. 123–142.

[23] N. P. Galatsanos, V. Z. Mesarovic, R. Molina, and A. K. Katsaggelos, "Hierarchical bayesian image restoration from partially known blurs," *IEEE Transactions on Image Processing*, vol. 9, no. 10, pp. 1784–1797, 2000.

[24] T. Schulz, "Multiframe blind deconvolution of astronomical images," *J. Opt. Soc. Am. A*, vol. 10, no. 5, pp. 1064–1073, May 1993.

[25] G. Harikumar and Y. Bresler, "Perfect blind restoration of images blurred by multiple filters: Theory and efficient algorithms," *IEEE Trans. Image Processing*, vol. 8, no. 2, pp. 202–219, Feb. 1999.

[26] M. I. Gurelli and C. L. Nikias, "EVAM: An eigenvector based algorithm for multichannel blind deconvolution of input colored signals," *IEEE Trans. Acous., Speech, Signal Processing*, vol. 43, pp. 134–149, 1995.

[27] G. Giannakis and R. Heath, "Blind identification of multichannel FIR blurs and perfect image restoration," *IEEE Trans. Image Processing*, vol. 9, no. 11, pp. 1877–1896, Nov. 2000.

[28] H.-T. Pai and A. Bovik, "On eigenstructure-based direct multichannel blind image restoration," *IEEE Trans. Image Processing*, vol. 10, no. 10, pp. 1434–1446, Oct. 2001.

[29] S. Pillai and B. Liang, "Blind image deconvolution using a robust GCD approach," *IEEE Trans. Image Processing*, vol. 8, no. 2, pp. 295–301, Feb. 1999.

[30] M. Haindl and S. Šimberová, "Model-based restoration of short-exposure solar images," in *Frontiers in Artificial Intelligence and Applications*, ser. Knowledge-Based Intelligent Engeneering Systems, L. Jain and R. Howlett, Eds., vol. 87. Amsterdam: Publisher IOS Press, 2002, pp. 697–706.

[31] G. Panci, P. Campisi, S. Colonnese, and G. Scarano, "Multichannel blind image deconvolution using the bussgang algorithm: Spatial and multiresolution approaches," *IEEE Trans. Image Processing*, vol. 12, no. 11, pp. 1324–1337, Nov. 2003.

[32] F. Šroubek and J. Flusser, "Multichannel blind iterative image restoration," *IEEE Trans. Image Processing*, vol. 12, no. 9, pp. 1094–1106, Sep. 2003.

[33] ——, "Multichannel blind deconvolution of spatially misaligned images," *IEEE Trans. Image Processing*, vol. 14, no. 7, pp. 874–883, Jul. 2005.

[34] F. Šroubek, G. Cristóbal, and J. Flusser, "A unified approach to superresolution and multichannel blind deconvolution," *IEEE Trans. Image Processing*, vol. 16, no. 9, pp. 2322–2332, Sep. 2007.

[35] N. Joshi, S. B. Kang, C. L. Zitnick, and R. Szeliski, "Image deblurring using inertial measurement sensors," *ACM Trans. Graph.*, vol. 29, pp. 30:1–30:9, July 2010. [Online]. Available: http://doi.acm.org/10.1145/1778765.1778767

[36] M. Ben-Ezra and S. Nayar, "Motion deblurring using hybrid imaging," in *Computer Vision and Pattern Recognition, 2003. Proceedings. 2003 IEEE Computer Society Conference on*, vol. 1, Jun. 2003, pp. I–657 – I–664 vol.1.

[37] M. Ben-Ezra and S. K. Nayar, "Motion-based motion deblurring," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 26, no. 6, pp. 689–698, Jun. 2004.

[38] Y.-W. Tai, H. Du, M. S. Brown, and S. Lin, "Image/video deblurring using a hybrid camera," in *CVPR*, 2008.

[39] O. Whyte, J. Sivic, A. Zisserman, and J. Ponce, "Non-uniform deblurring for shaken images," in *Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on*, Jun. 2010, pp. 491 –498.

[40] A. Gupta, N. Joshi, C. L. Zitnick, M. Cohen, and B. Curless, "Single image deblurring using motion density functions," in *Proceedings of the 11th European conference on Computer vision: Part I*, ser. ECCV'10. Berlin, Heidelberg: Springer-Verlag, 2010, pp. 171–184. [Online]. Available: http://portal.acm.org/citation.cfm?id=1886063.1886078

[41] M. Hirsch, C. J. Schuler, S. Harmeling, and B. Scholkopf, "Fast removal of non-uniform camera shake," in *Proc. IEEE Int Computer Vision (ICCV) Conf*, 2011, pp. 463–470. [Online]. Available: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=6126276

[42] M. Šorel and J. Flusser, "Space-variant restoration of images degraded by camera motion blur," *IEEE Transactions on Image Processing*, vol. 17, no. 2, pp. 105–116, Feb. 2008.

[43] P. Favaro, M. Burger, and S. Soatto, "Scene and motion reconstruction from defocus and motion-blurred images via anisothropic diffusion," in *ECCV 2004, LNCS 3021, Springer Verlag, Berlin Heidelberg*, T. Pajdla and J. Matas, Eds., 2004, pp. 257–269.

[44] M. Šorel and F. Šroubek, "Space-variant deblurring using one blurred and one underexposed image," in *Proceedings of the IEEE 16th International Conference on Image Processing ICIP 2009*. IEEE, 2009.

[45] ——, "Restoration in the presence of unknown spatially varying blur," in *Image Restoration: Fundamentals and Advances*, G. Bahadir and L. Xin, Eds. CRC Press, 2012.

[46] T. L. Williams, *The optical transfer function of imaging systems.* Institute of Physics Publishing, London, 1999.

[47] N. Joshi, R. Szeliski, and D. J. Kriegman, "PSF estimation using sharp edge prediction," *Computer Vision and Pattern Recognition, IEEE Computer Society Conference on*, pp. 1–8, 2008.

[48] M. Tico, M. Trimeche, and M. Vehvilainen, "Motion blur identification based on differently exposed images," in *Proc. IEEE Int. Conf. Image Processing*, 2006, pp. 2021–2024.

[49] L. Yuan, J. Sun, L. Quan, and H.-Y. Shum, "Image deblurring with blurred/noisy image pairs," in *SIGGRAPH '07: ACM SIGGRAPH 2007 papers*. New York, NY, USA: ACM, 2007, p. 1.

[50] F. Šroubek, M. Šorel, I. Horackova, and J. Flusser, "Patch-based blind deconvolution with parametric interpolation of convolution kernels," in *Proc. IEEE International Conference on Image Processing*, 2013, pp. 1–4.

[51] A. Chakrabarti, T. Zickler, and W. T. Freeman, "Analyzing spatially-varying blur," in *CVPR'10*, San Francisco, CA, USA, Jun. 2010, pp. 2512–2519.

[52] S. Dai and Y. Wu, "Motion from blur," in *Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on*, Jun. 2008, pp. 1 –8.

[53] R. Liu, Z. Li, and J. Jia, "Image partial blur detection and classification," in *Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on*, Jun. 2008, pp. 1 –8.

[54] A. Levin, "Blind motion deblurring using image statistics," in *Advances in Neural Information Processing Systems (NIPS)*, 2006, pp. 841–848.

[55] H. Takeda and P. Milanfar, "Removing motion blur with space–time processing," *IEEE Transactions on Image Processing*, vol. 20, no. 10, pp. 2990–3000, 2011.

[56] L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D*, vol. 60, pp. 259–268, 1992.

[57] D. Mumford and J. Shah, "Optimal approximation by piecewise smooth functions and associated variational problems," *Comm. Pure Appl. Math.*, vol. 42, pp. 577–685, 1989.

[58] D. Geman and G. Reynolds, "Constrained restoration and the recovery of discontinuities," *IEEE Trans. Pattern Anal.*, vol. 14, no. 3, pp. 367–383, Mar. 1992.

[59] B. Zitová and J. Flusser, "Image registration methods: A survey," *Image and Vision Computing*, vol. 21, pp. 977–1000, 2003.